Chiral extrapolation of lattice QCD results

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Nucleon couples strongly to pions in QCD

- Goldberger-Treiman relation [1958]: In the chiral limit

$$g_{\pi NN} = \frac{g_A M_N}{f_\pi}$$

- Accurate to ~3% at physical point

$$g_{\pi NN} \sim 13.3 \qquad \frac{g_A M_N}{f_\pi} \sim 12.9$$

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$$m_\pi^2 \propto m_q$$

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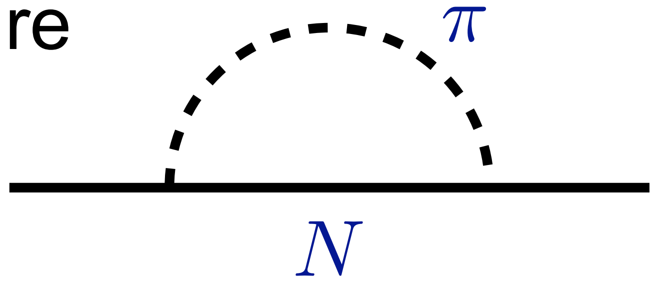
Field-theoretic consequences for nucleon structure



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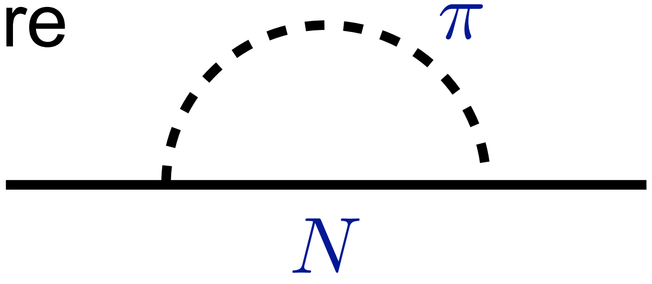
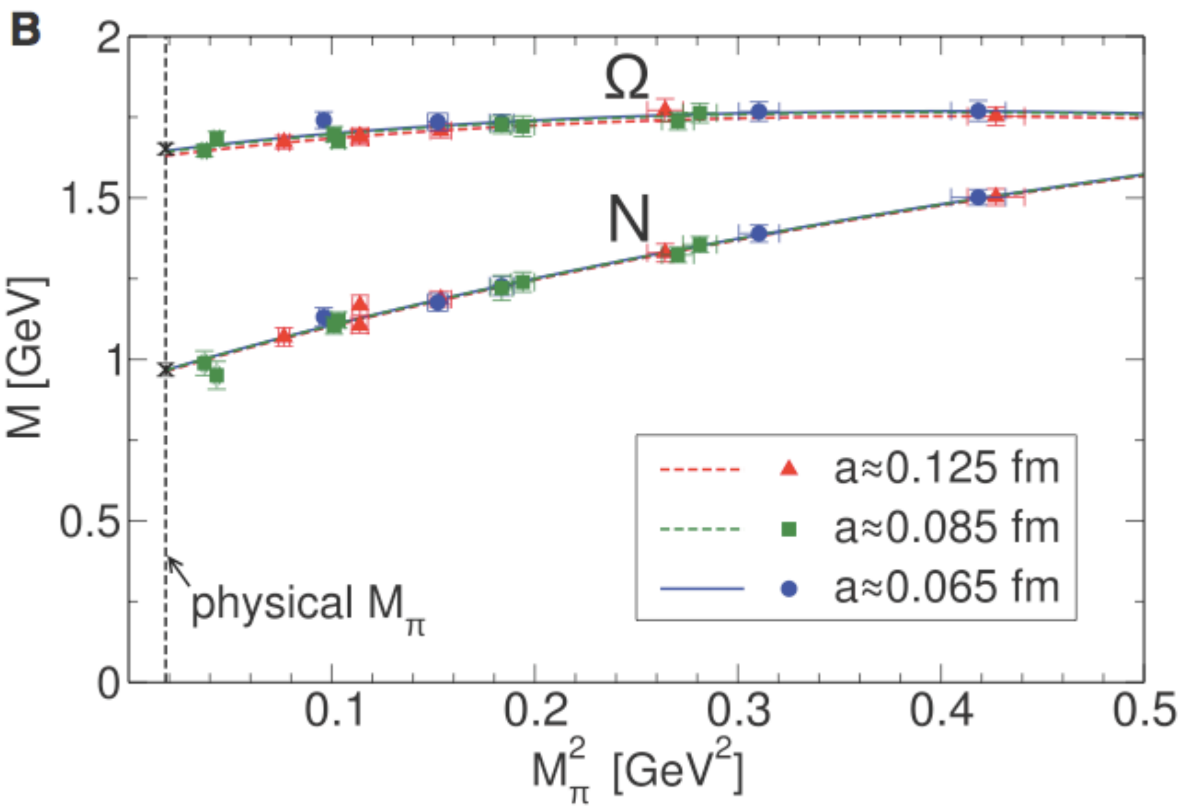


Nucleon properties exhibit nonanalytic expansion about chiral limit

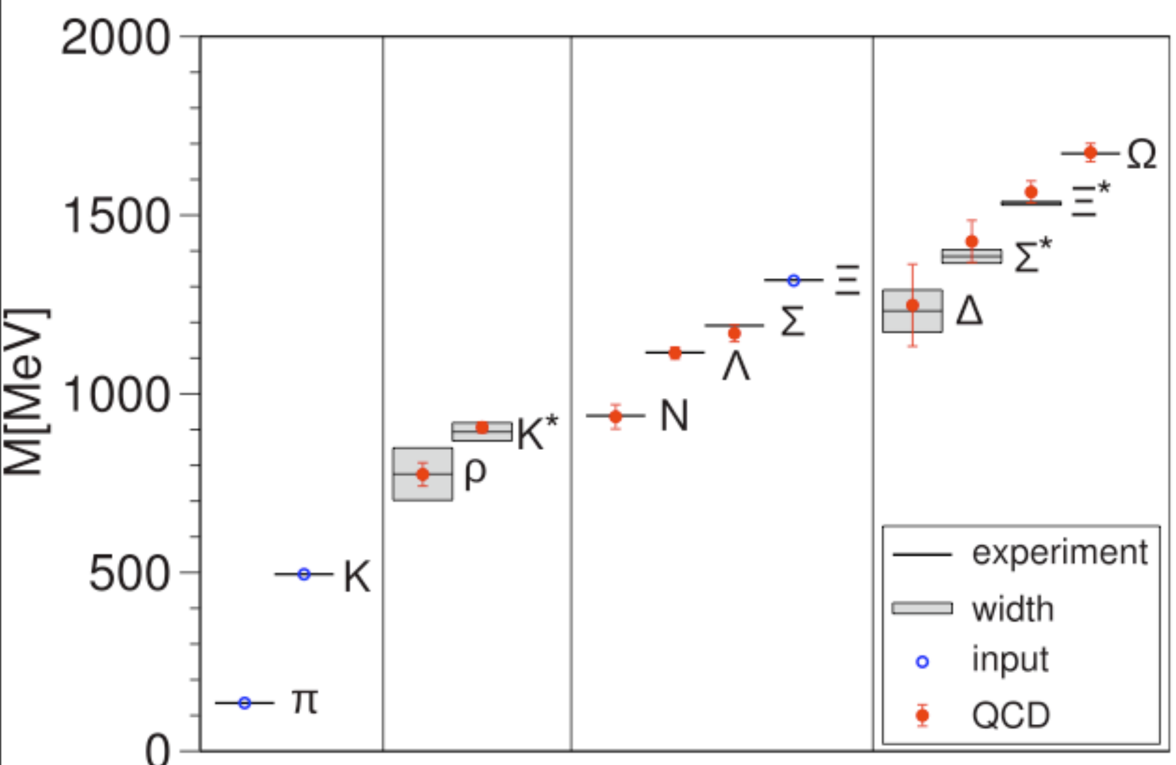
eg. $$M_N^{LNA} \sim -\frac{3}{32\pi}\frac{g_A^2}{f_\pi^2} m_\pi^3$$

Science

Lattice QCD: Entering the chiral regime



- BMW, *Science* 322, 1224 (2008)
  - * Multiple lattice spacing
  - * Multiple volumes
  - * Variant strange-quark mass runs
  - * mpi ~ 190 MeV



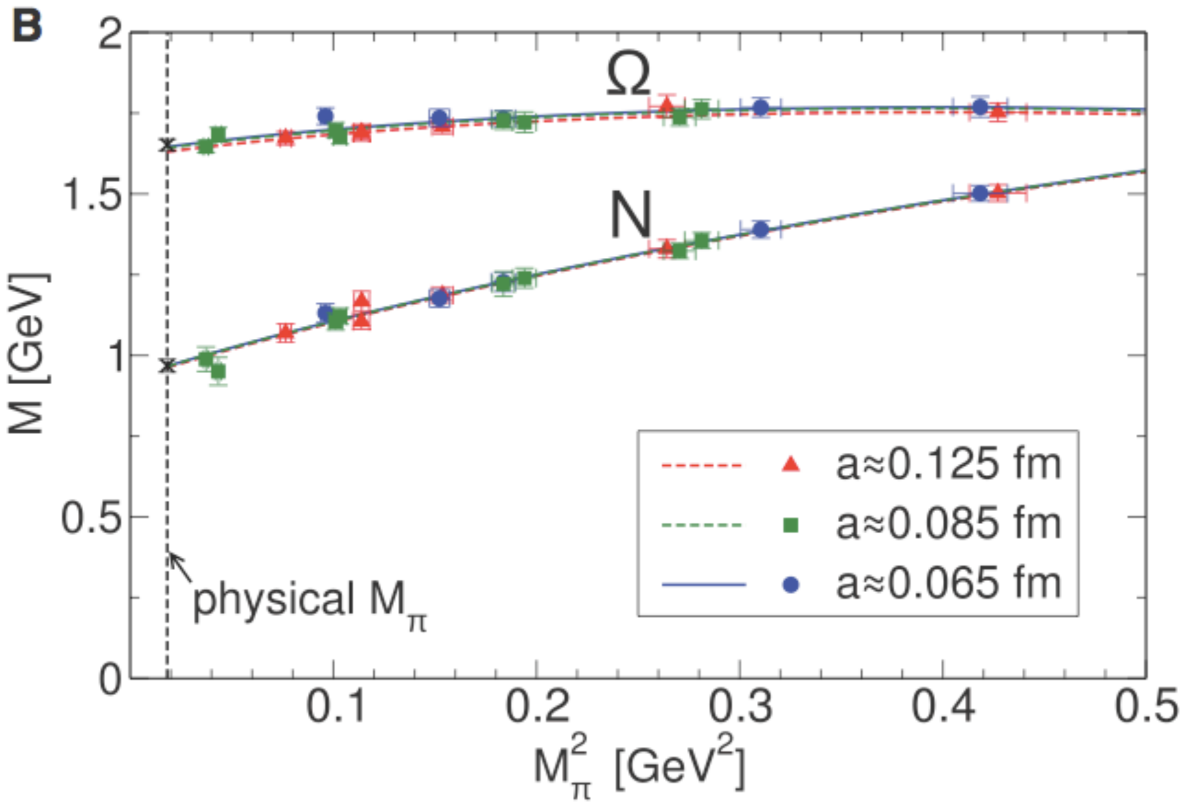
- Empirical chiral extrapolation

Chiral extrapolation

- Statistical average over many different fits (432 total fits)
  * 3 pion mass ranges
  * 2 scale setting procedures

$$r_X = \frac{m_X}{m_\Xi}$$

  * 2 pion mass extrapolation strategies



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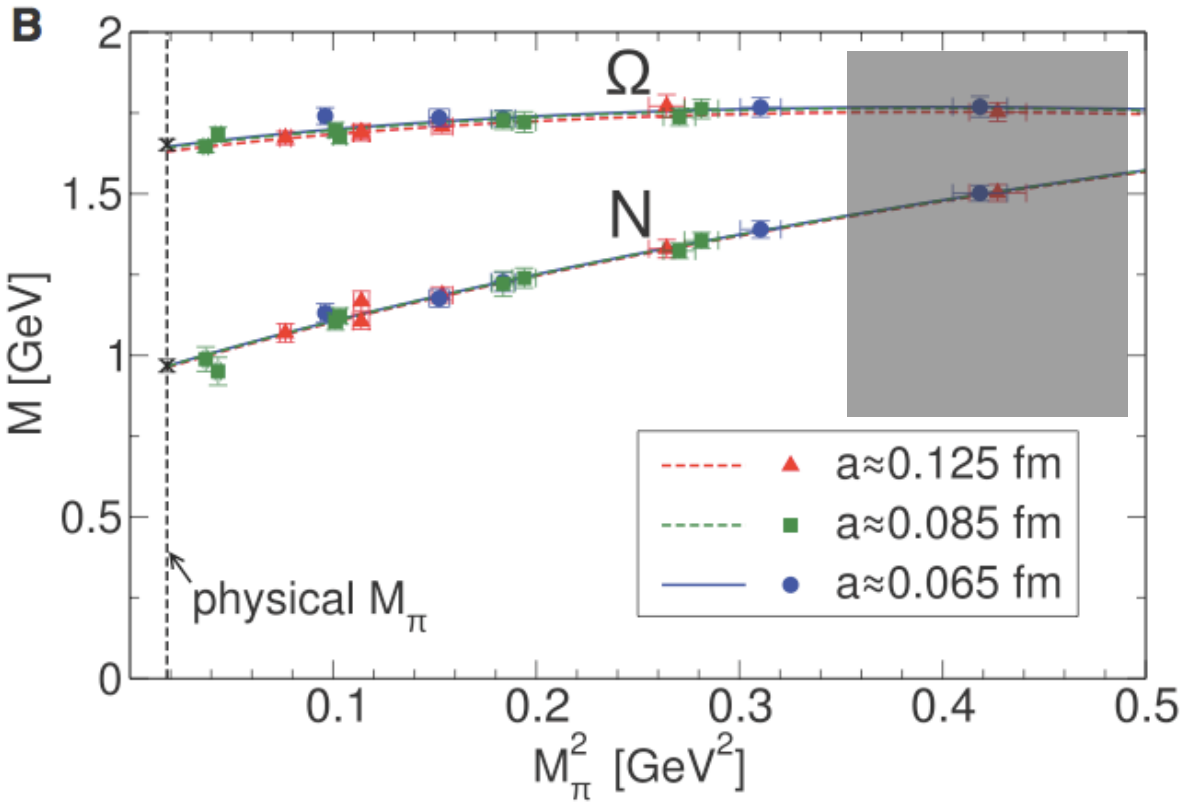

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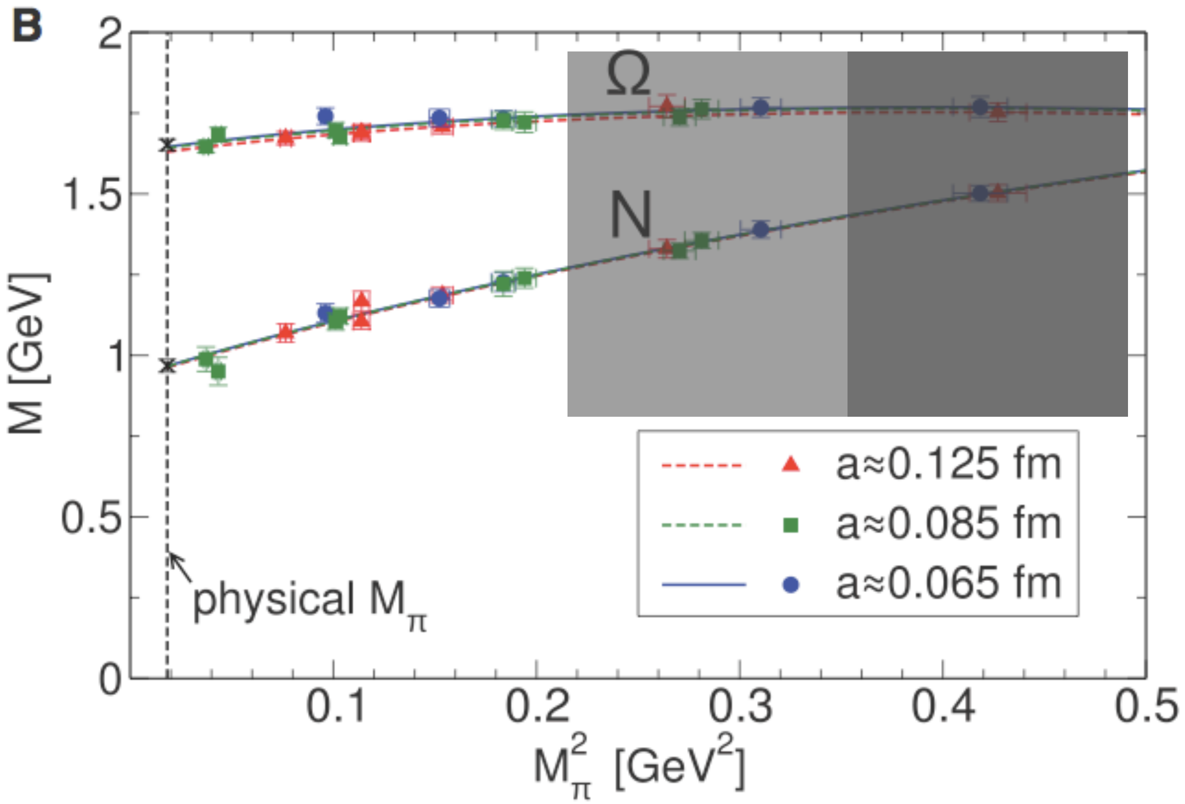

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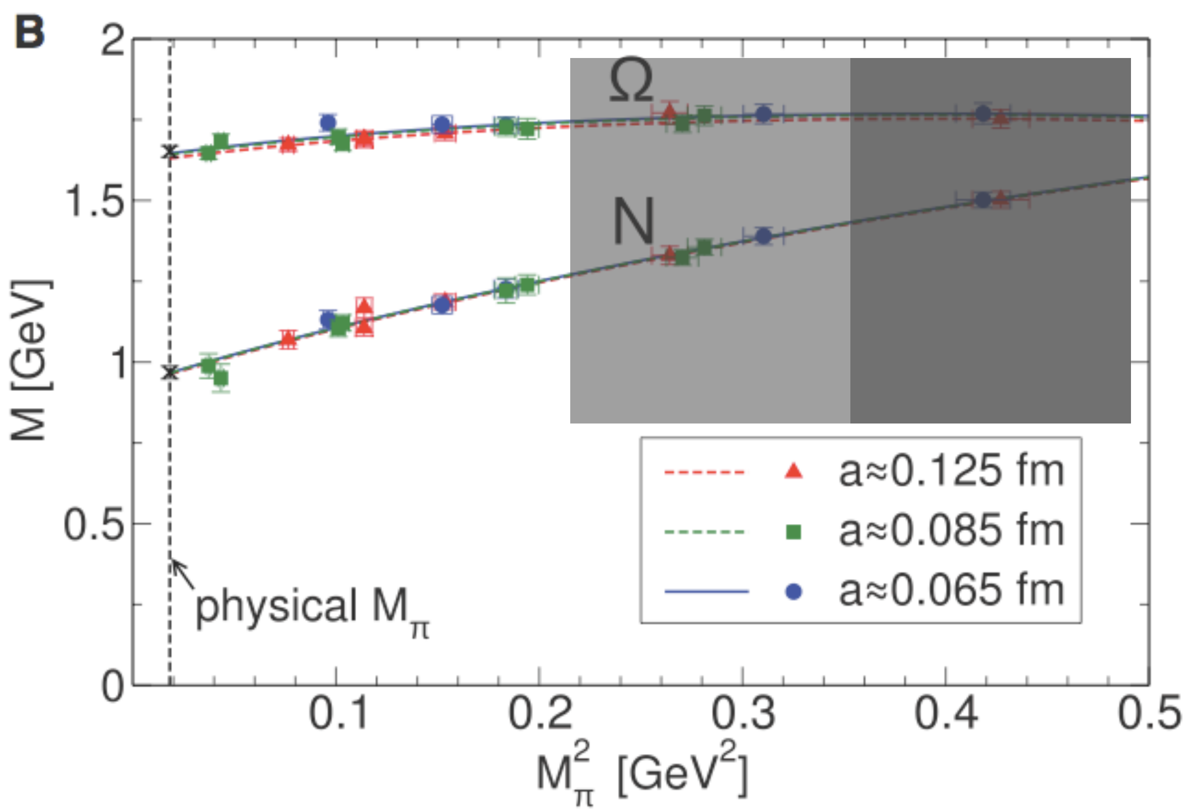

Chiral $$r_N = r_N^{ref} + \alpha_N r_\pi^2 + \alpha'_N r_\pi^3 + \beta_N \left[r_K^2 - (r_K^{ref})^2 \right]$$

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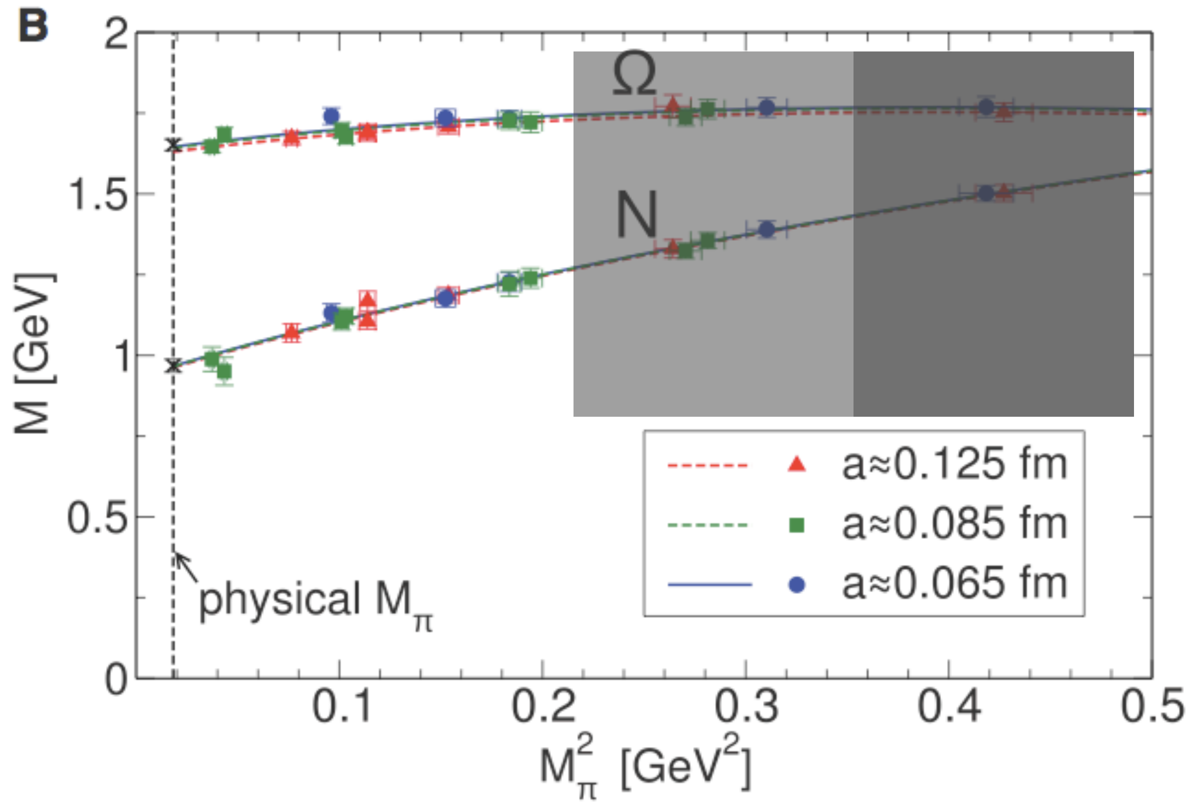

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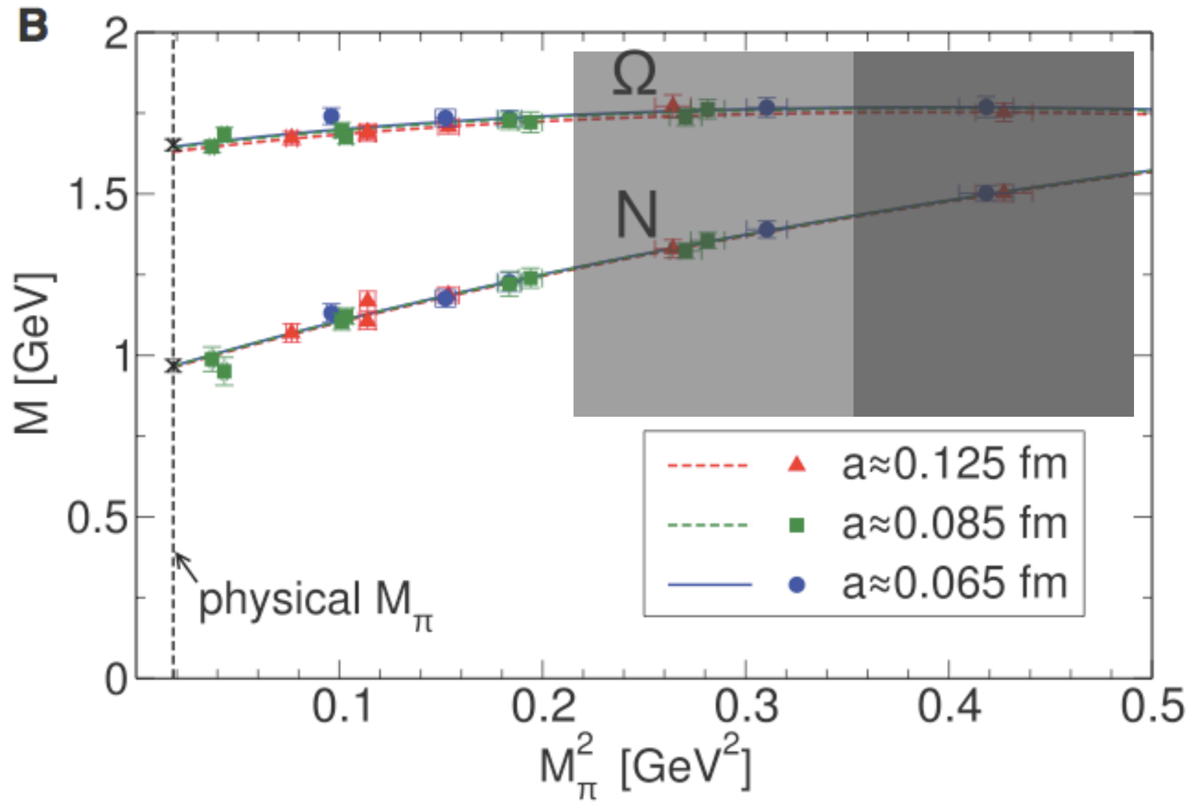

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“Chiral” fit

- Expectation of QCD

$$M_N^{LNA} \sim -\frac{3}{32\pi}\frac{g_A^2}{f_\pi^2} m_\pi^3 \sim (-5.6\,\mathrm{GeV}^{-2}) m_\pi^3$$

$$\left(\frac{M_N}{M_\Xi}\right)^{LNA} \sim (-5.4\,\mathrm{GeV}^{-2}) m_\pi^3$$

- *Lightest* cut on lattice results

$$\alpha'_N \sim -4.5(3.2)(0.5)\,\mathrm{GeV}^{-2}$$

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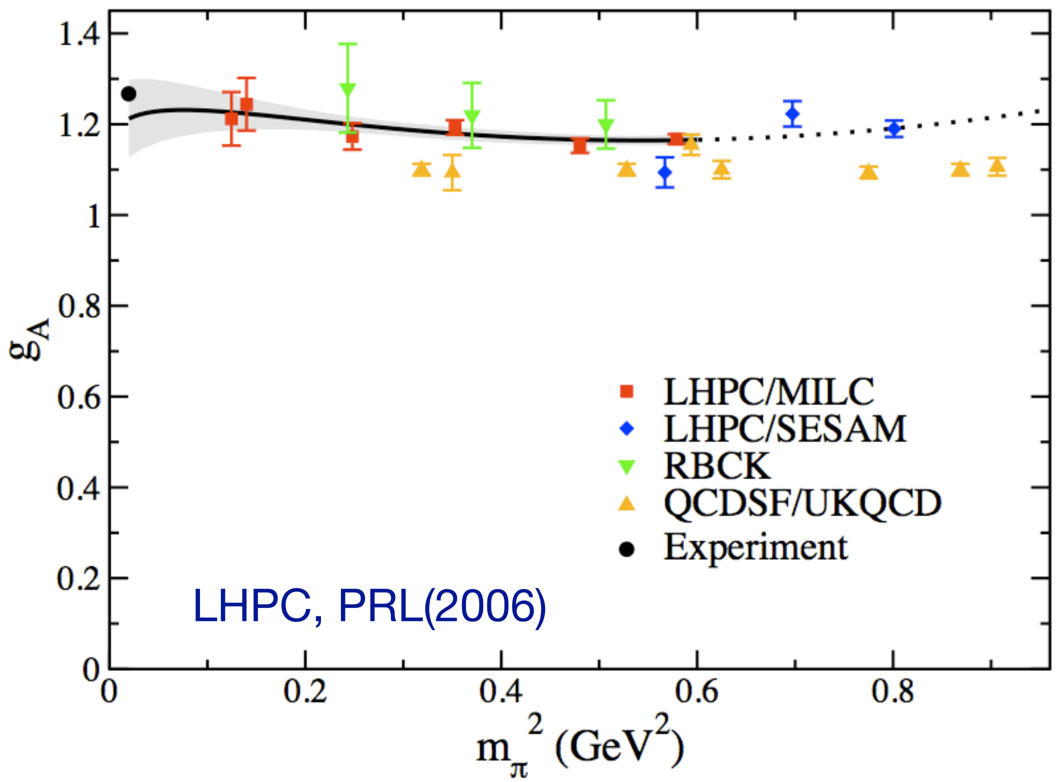
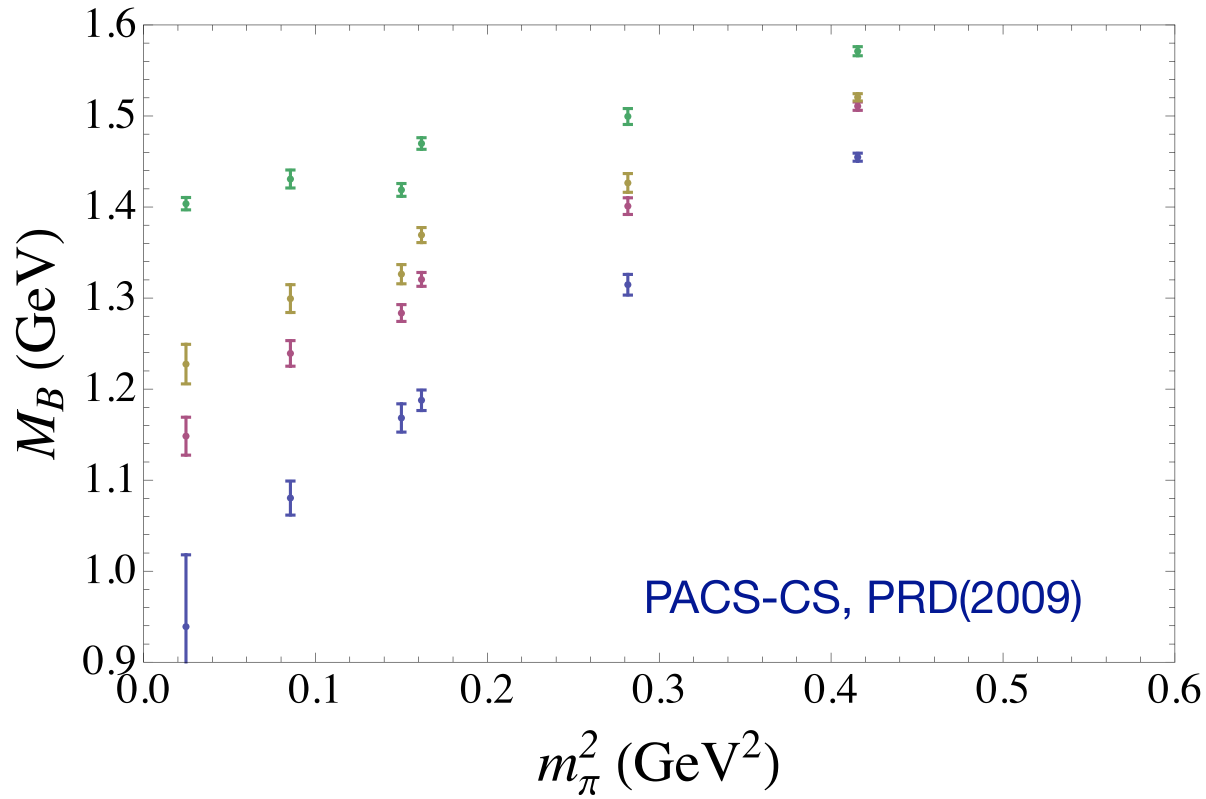


Statistical challenges

- Baryon signal-to-noise degradation

$$\frac{\text{Signal}}{\text{Noise}} \propto \exp\left\{-(M_N - \frac{3}{2}m_\pi)t\right\}$$

NPLQCD



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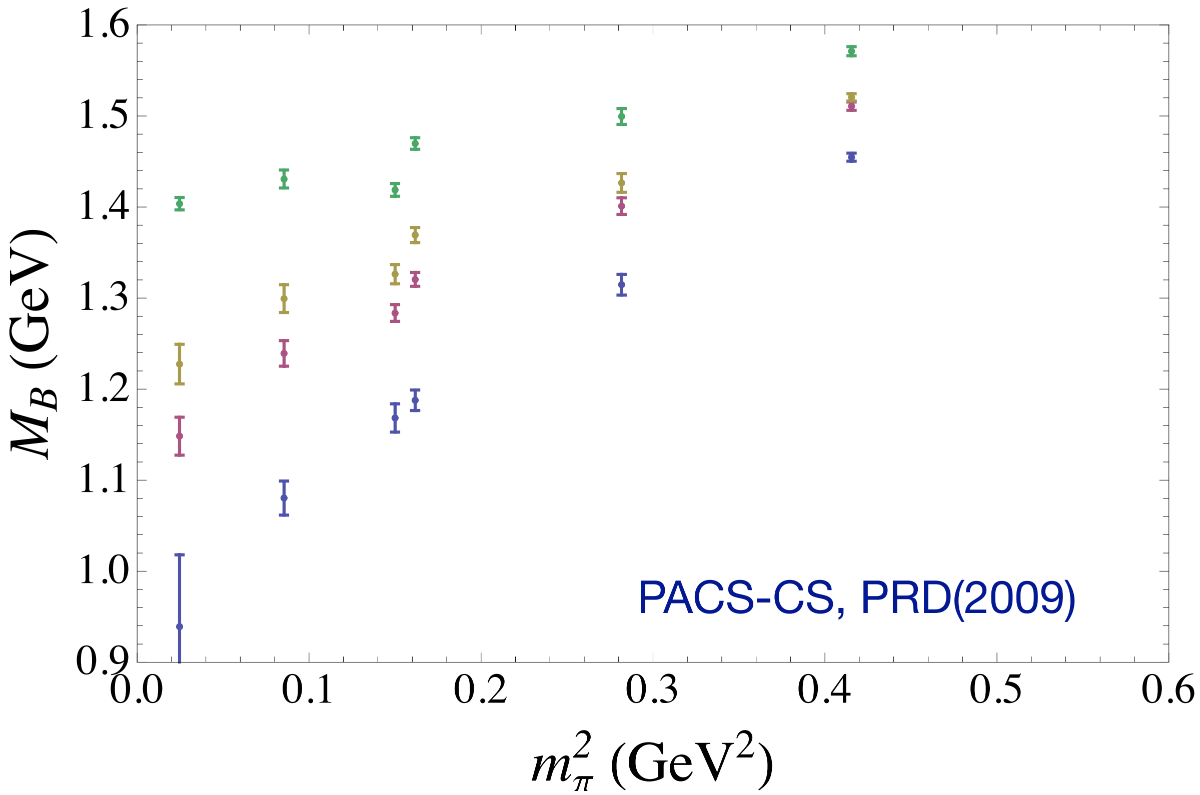
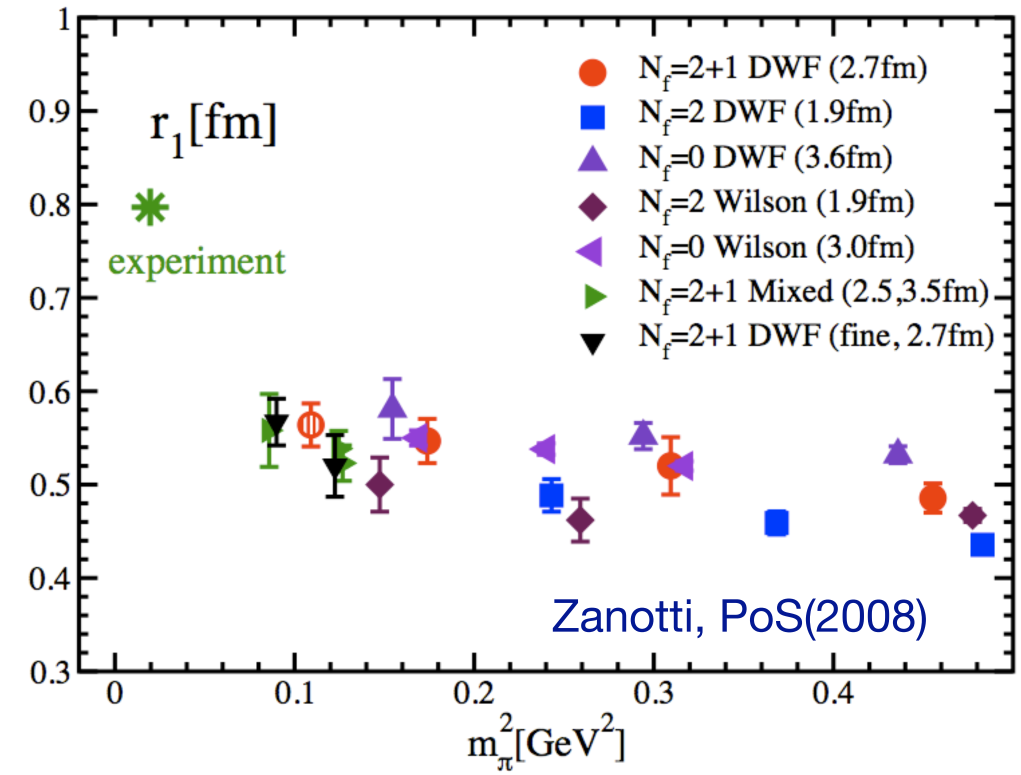


Structure properties can be rapidly varying

$$\langle r^2 \rangle_1^V \sim -\frac{2}{(4\pi f_\pi)^2}(5g_A^2+1)\log\frac{m_\pi}{\mu}$$



Chiral effective field theory offers a method to dramatically reduce statistical uncertainties

Can correlate seemingly uncorrelated observables

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- Consider octet baryon magnetic moments

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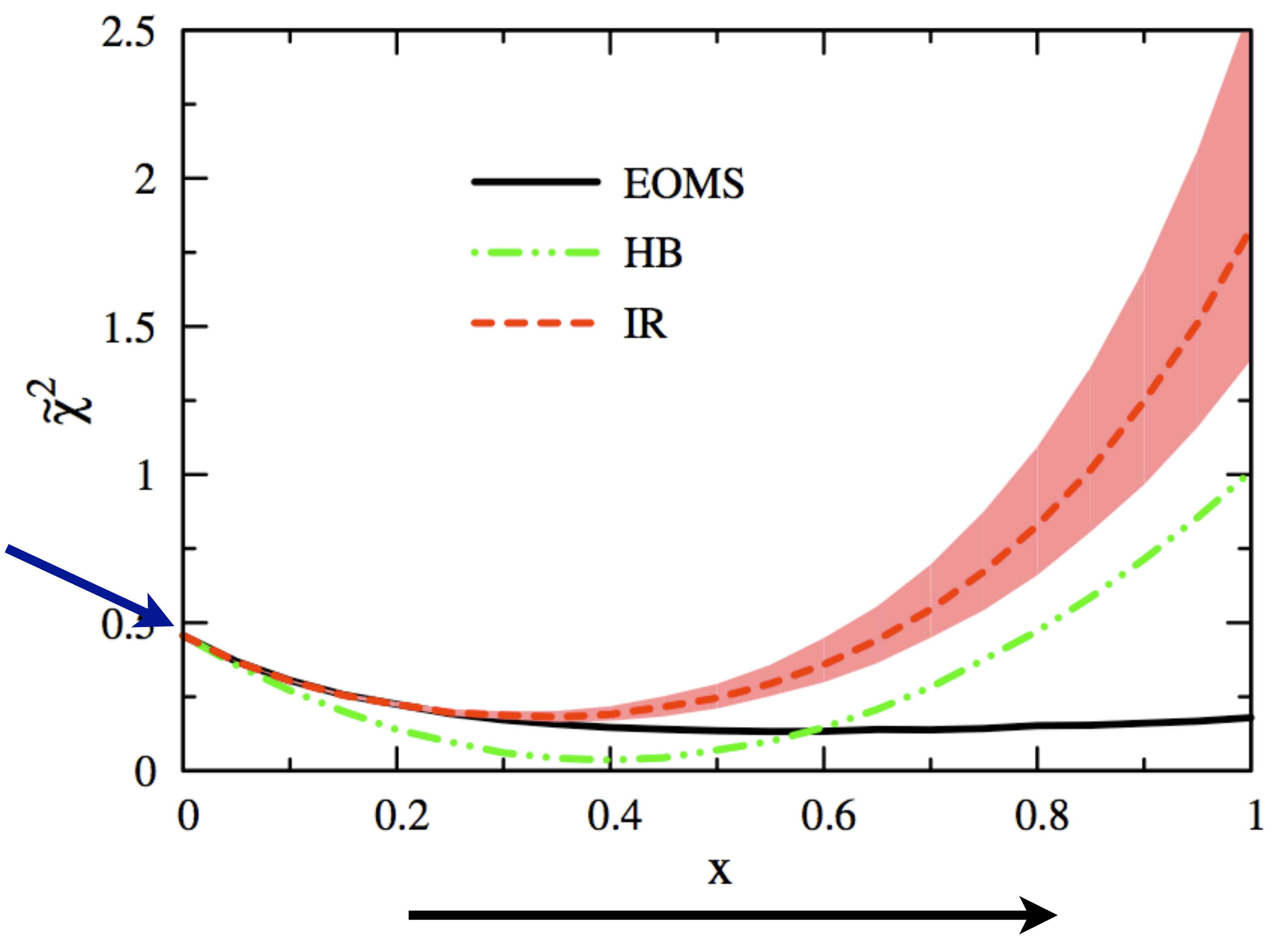
Extended-on-mass-shell (EOMS): improved expansion — Geng *et al*., PRL(2008)

One loop correction

Geng *et al*., PRL(2008)

No loop: CG

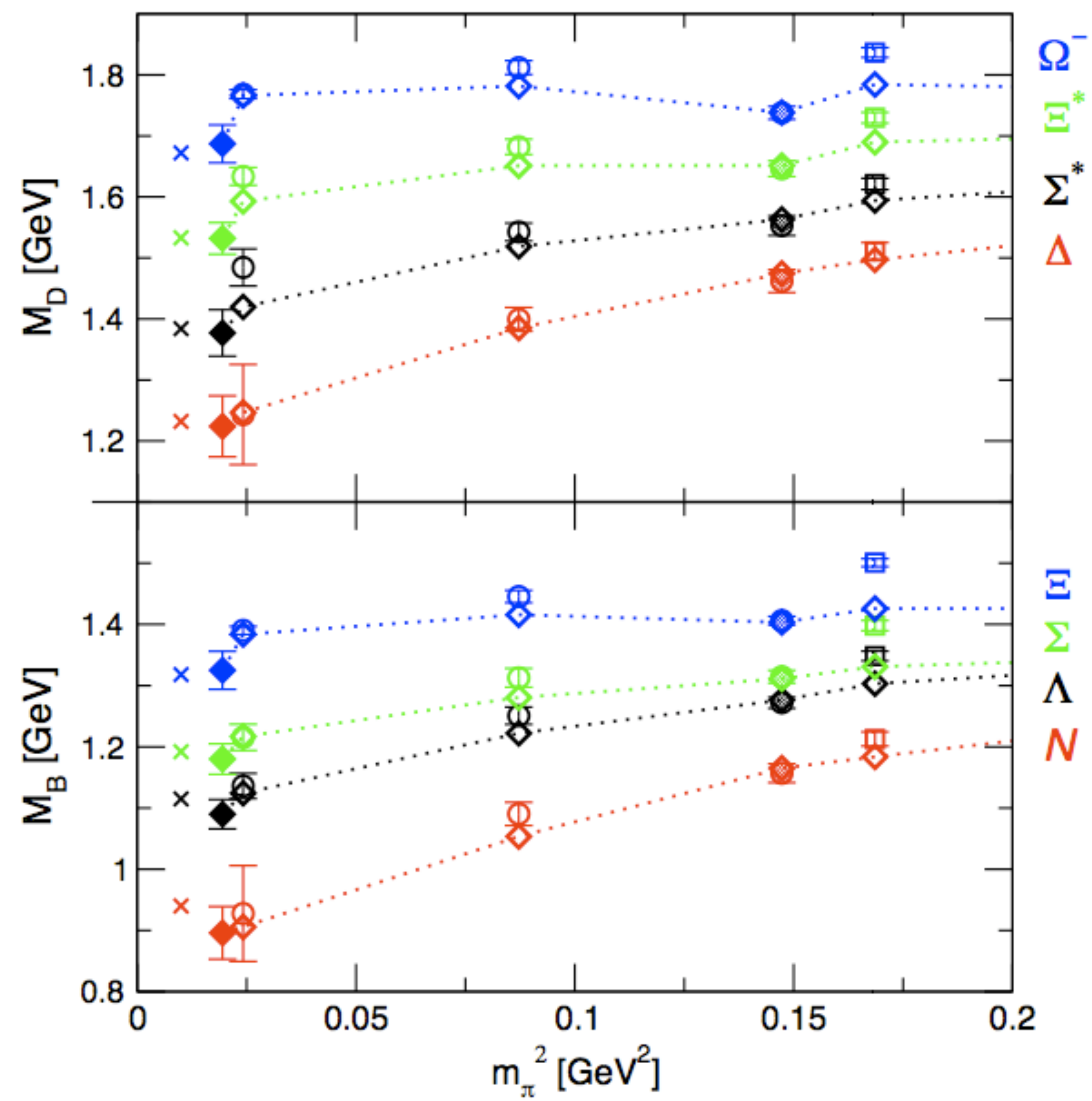


Evolve meson masses
from 0 to physical

SU(3) for lattice QCD

- Fits to octet and decuplet baryon masses in EOMS

	GMO	HB	Covariant
$\chi^2_{\rm d.o.f.}$	0.60	9.3	2.2
$\bar{\chi}^2_{\rm d.o.f.}$ w/expt.	4.3	36.4	2.8

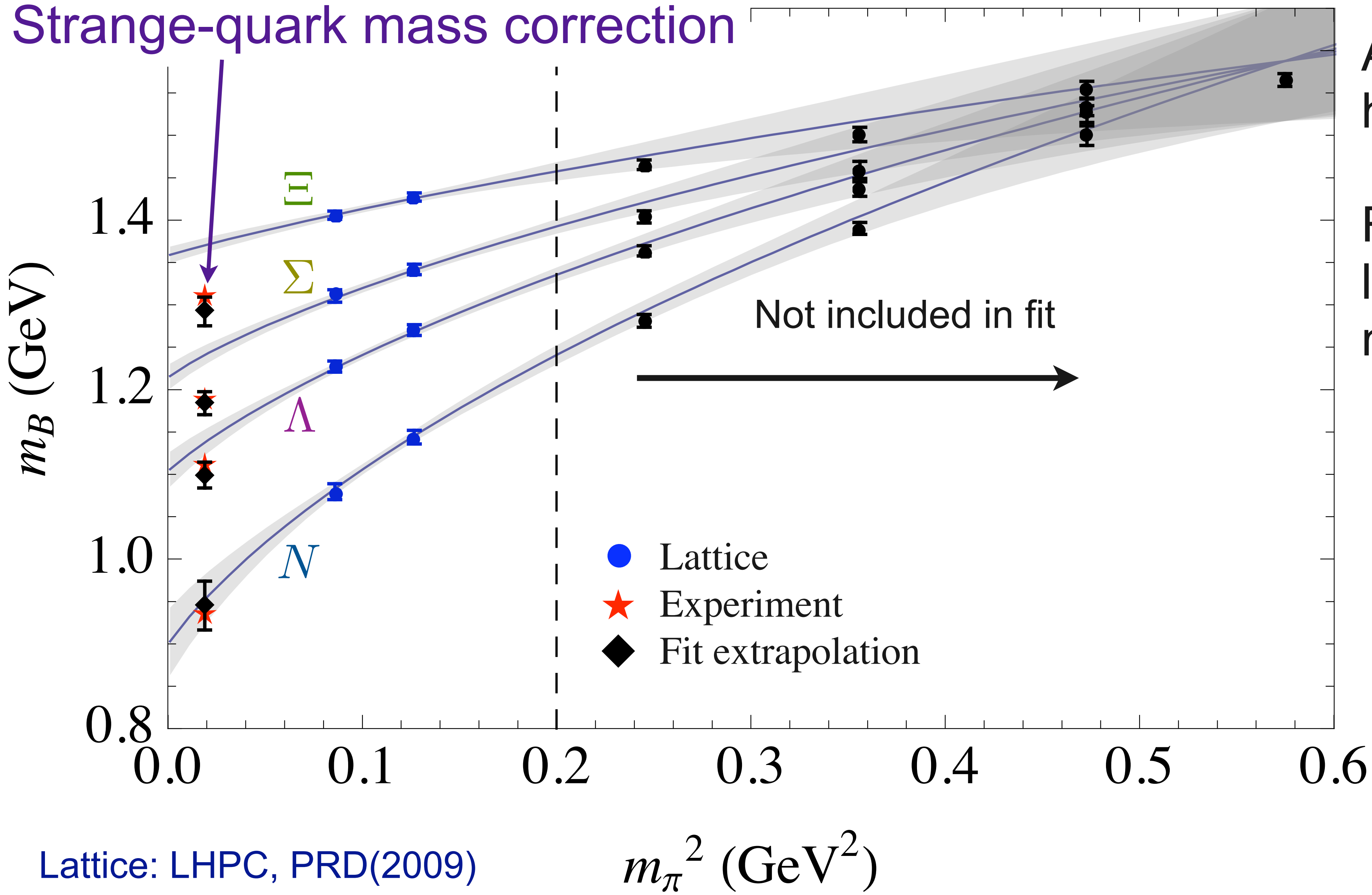


Martin-Camalich *et al*., arXiv:1003.1929

Finite-range regularisation (FRR)

$m_s^{latt} \sim 1.3 m_s^{phys}$

Strange-quark mass correction



Accurate prediction of heavier simulation data

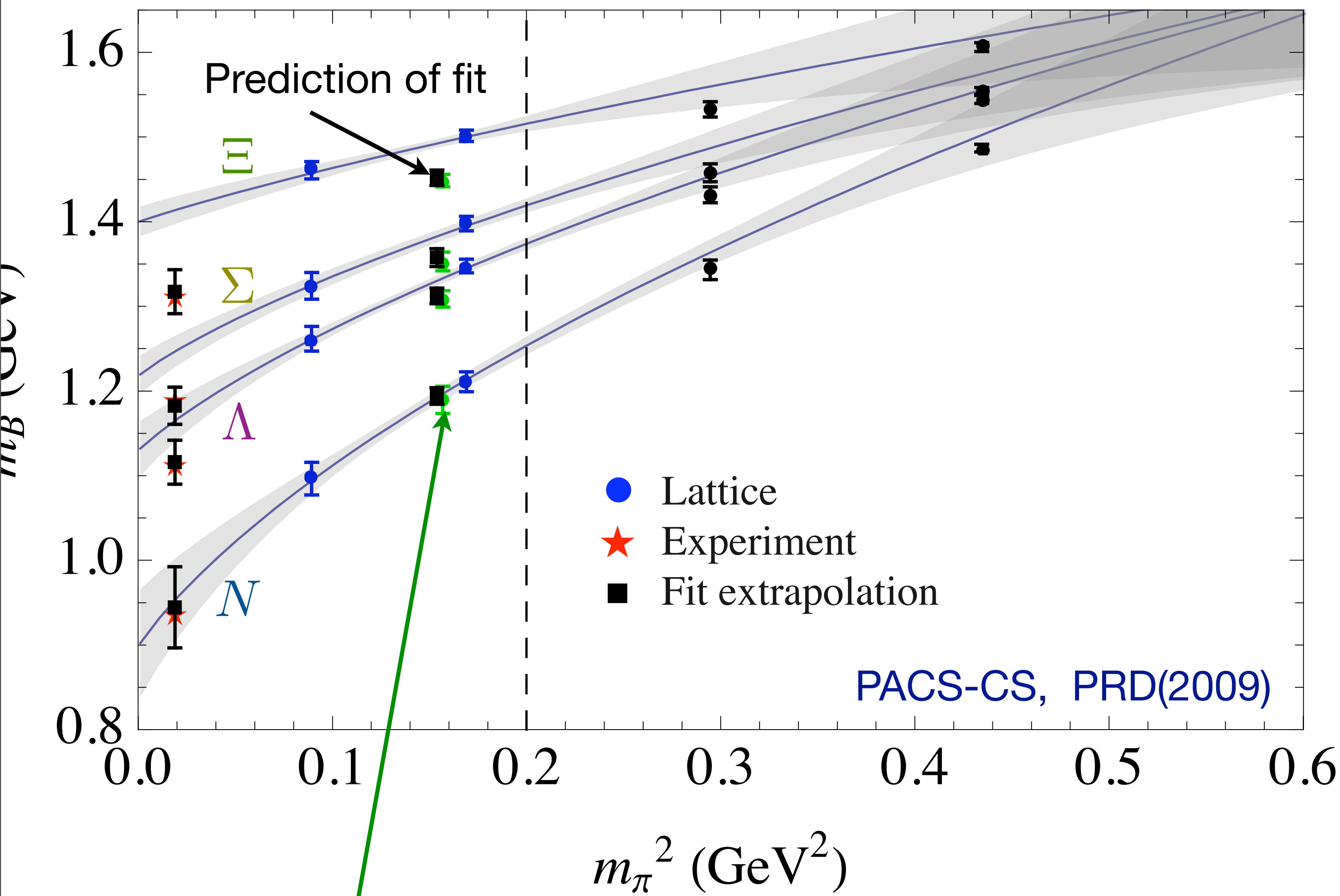
Reliable correction for lattice simulation quark mass

Lattice: LHPC, PRD(2009)

RDY & Thomas, PRD(2010)

PACS-CS fits

PACS-CS: 2+1-flavour simulation; different action discretization to LHPC



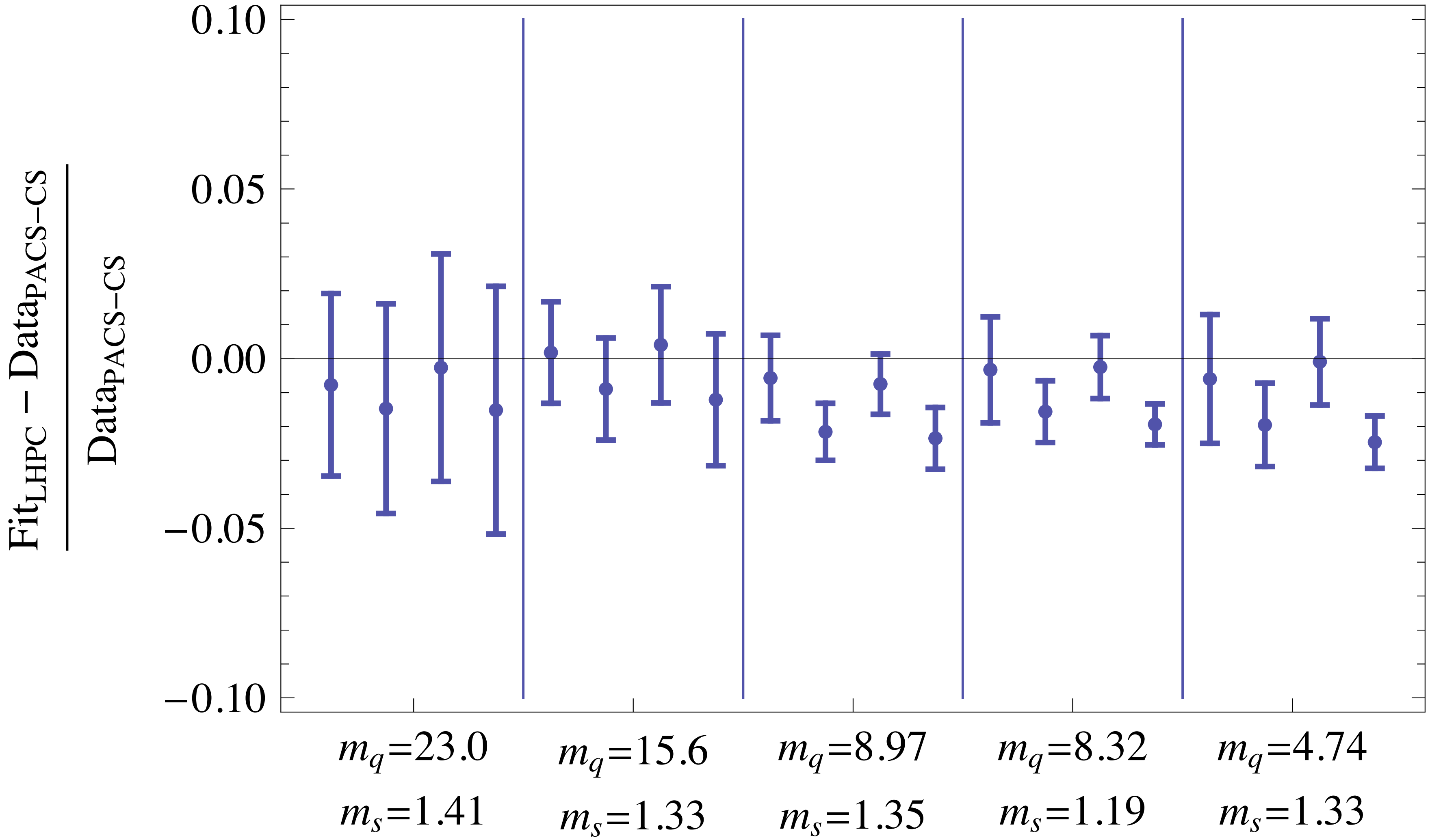
Correction in strange quark mass demonstrated to be reliable against numerical simulation

As for LHPC, excellent agreement with observed spectrum

PACS-CS have an additional run with a different strange quark mass

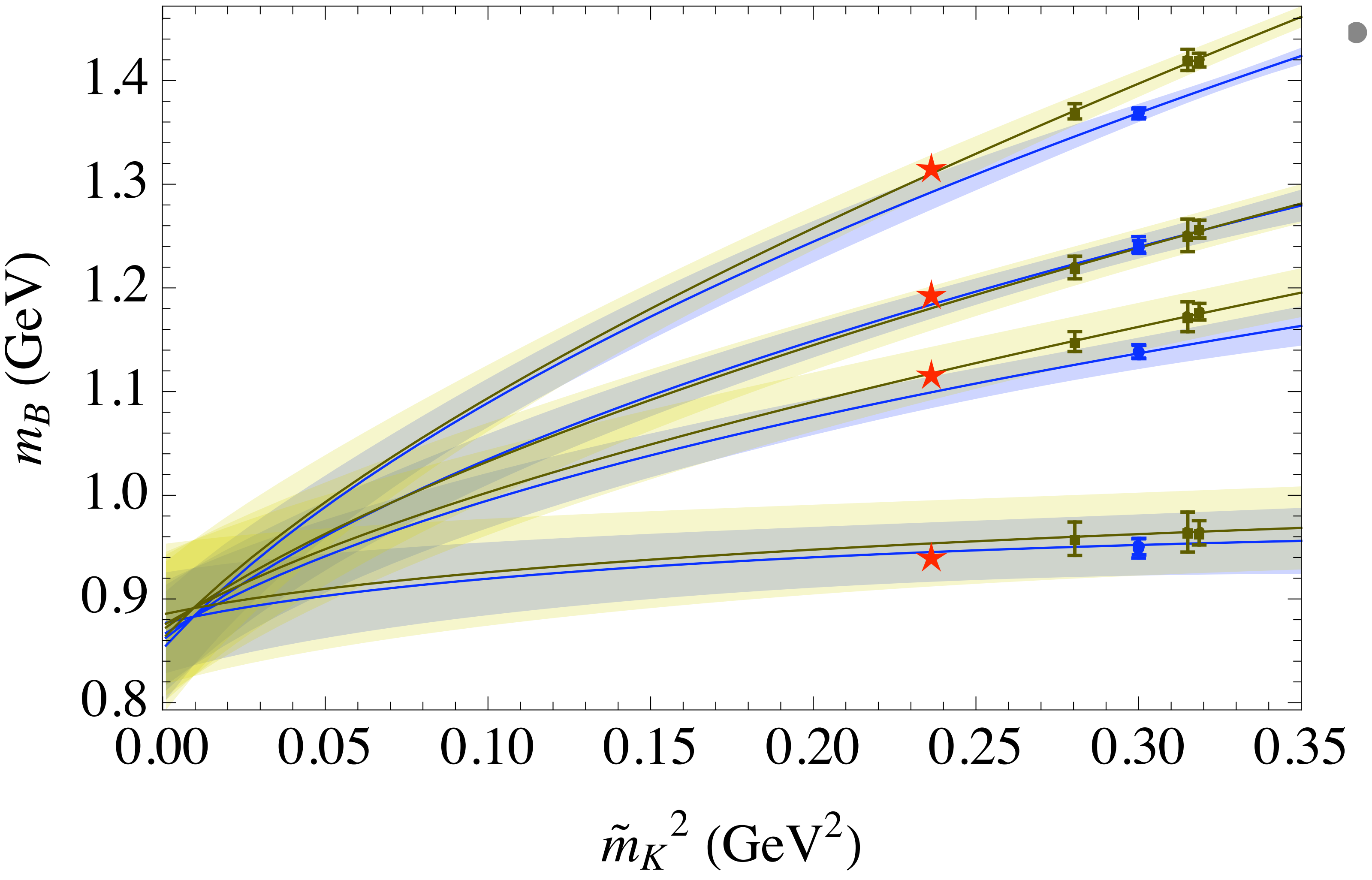
RDY & Thomas, PRD(2010)

“Predict” PACS-CS from LHPC



Discretization errors appear at 1–2% level

Strange-quark mass dependence



- Strange quark "extrapolation" at physical pion mass

Quantifying uncertainties

Nucleon Mass (GeV)

Discretisation

LHPC	0.945 ± 0.029
PACS-CS	0.954 ± 0.042

Extrapolated baryon masses and fit parameters (LECs) in agreement

Regulator

Dipole	0.9410
Sharp	0.9452

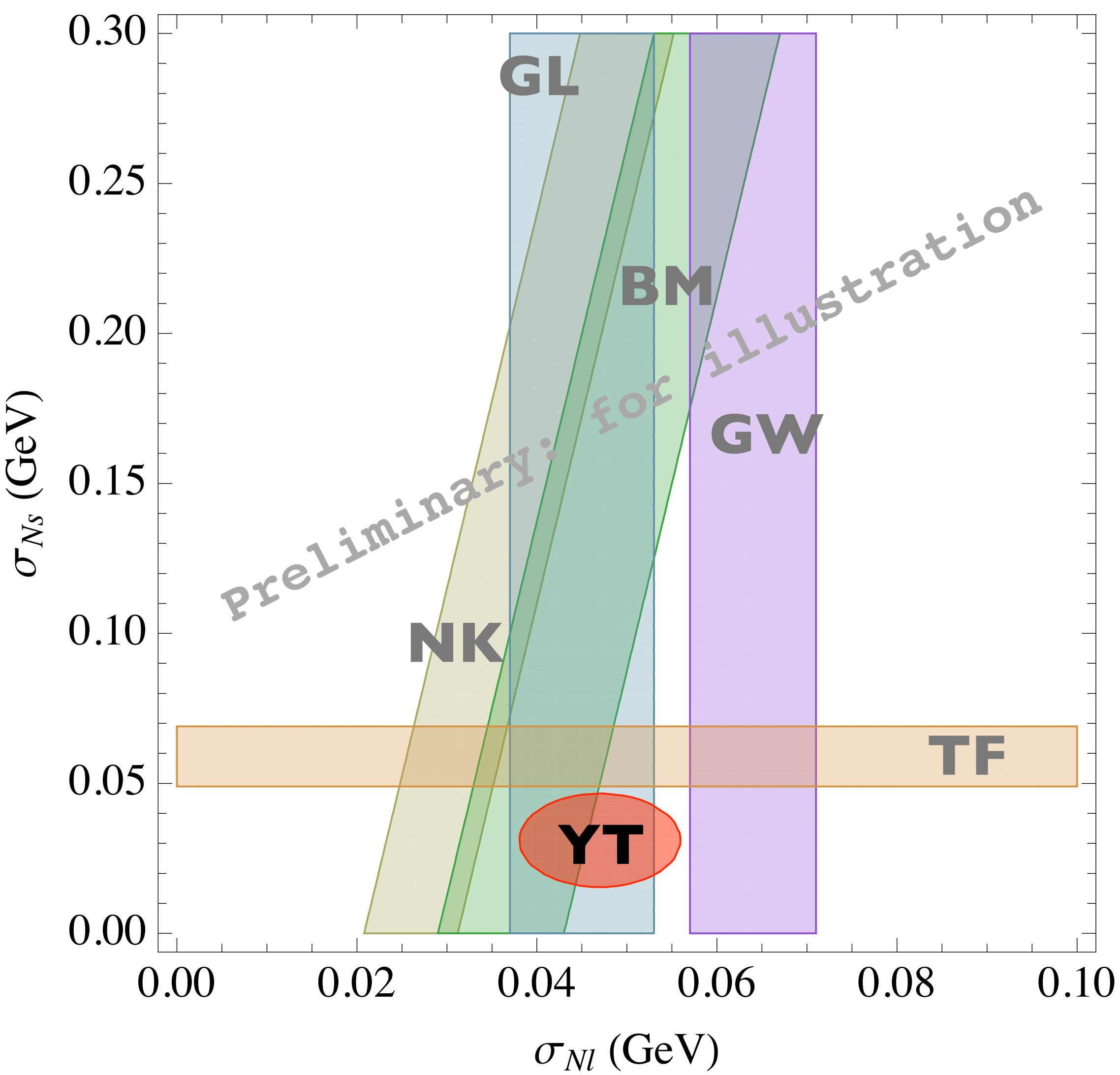
Small dependence on choice of regulator — similarly for other functional forms (monopole, Gaussian)

Source	MeV
Statistical	**23.6**
Discretisation	**4.2**
Model	**3.1**
Regulator	2.1
f_π (5%)	0.7
F (15%)	1.3
D (15%)	1.3
$\mathcal{C}$ (15%)	0.9
Δ_{10-8} (15%)	0.4

Baryon Sigma Terms

$$\bar{\sigma}_{Bq} = \frac{m_q}{M_B}\frac{\partial M_B}{\partial m_q}$$

	N	Λ	Σ	Ξ
$\bar{\sigma}_{Bl}$	**0.050(9)(1)(3)**	**0.028(4)(1)(2)**	**0.0212(27)(1)(17)**	**0.0100(10)(0)(4)**
$\bar{\sigma}_{Bs}$	**0.033(16)(4)(2)**	**0.144(15)(10)(2)**	**0.187(15)(3)(4)**	**0.244(15)(12)(2)**



πN Sigma Term (Expt):
GL: Gasser & Leutwyler (1991)
GW: Pavan et al. (2001)

Octet Masses & Breaking:
Gasser (1981)
NK: Nelson & Kaplan (1987)
BM: Borasoy & Meissner (1997)

3-flavour Lattice QCD:
YT: Young & Thomas (2009)
TF: Toussaint & Freeman (2009)

We determine precisely *both* the light and strange quark sigma terms

Spin-independent neutralino cross sections

- Ellis, Olive & Savage, PRD(2008)
  * Constrained Minimal Supersymmetric Standard Model (CMSSM)
  * Neutralino as dark matter candidate
  * Scalar contact interaction

$$\mathcal{L}_{SI} = \sum_i \alpha_{3i} \bar{\chi}\chi \bar{q}_i q_i$$

$$\sigma^p_{SI} \propto |f_p|^2$$

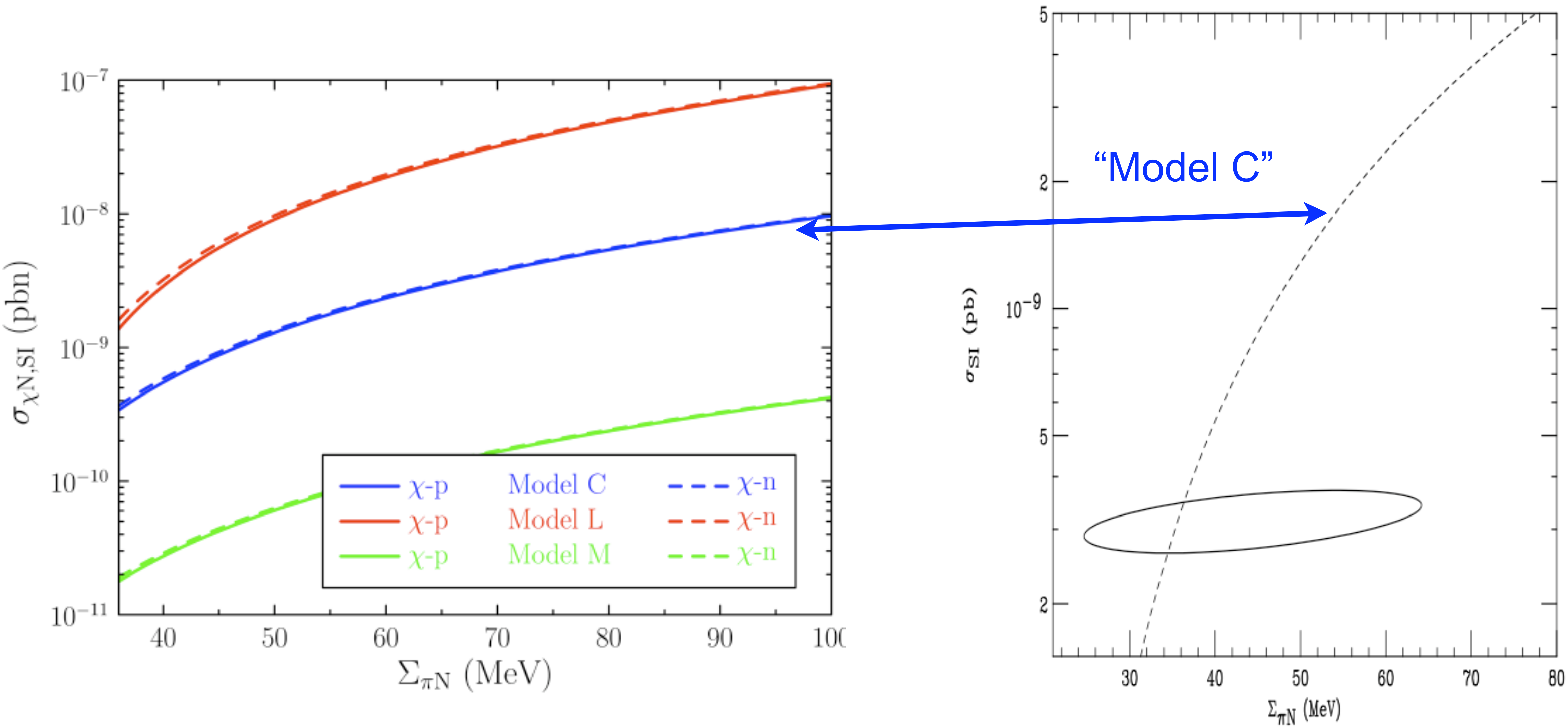
$$\frac{f_p}{M_p} = \sum_{q=u,d,s} \bar{\sigma}_{pq} \frac{\alpha_{3q}}{m_q} + \frac{2}{27} f^p_{TG} \sum_{q=c,b,t} \frac{\alpha_{3q}}{m_q}$$

$$f^p_{TG} = 1 - \sum_{q=u,d,s} \bar{\sigma}_{pq}$$

Trace anomaly:
Shifman, Vainstein & Zakharov, PLB(1978)

Uncertainty dominated by knowledge of light-quark sigma terms

Updated cross sections for benchmark models



Ellis, Olive & Savage

Giedt, Thomas & RDY, PRL(2009)

Strong dependence on sigma term from poorly known strangeness

Tremendous advance in precision from new lattice QCD results

Nuclear physics & lattice QCD can help discriminate supersymmetry scenarios

Remarks

- Lattice calculations are reaching the physical pion mass as we speak
- Remains a strong need for chiral approaches to extract the most physics
- Expansions should not be limited by the worst way of formulating
- SU(3) can be used effectively for baryons